Optimal placement for coded caching at the edge

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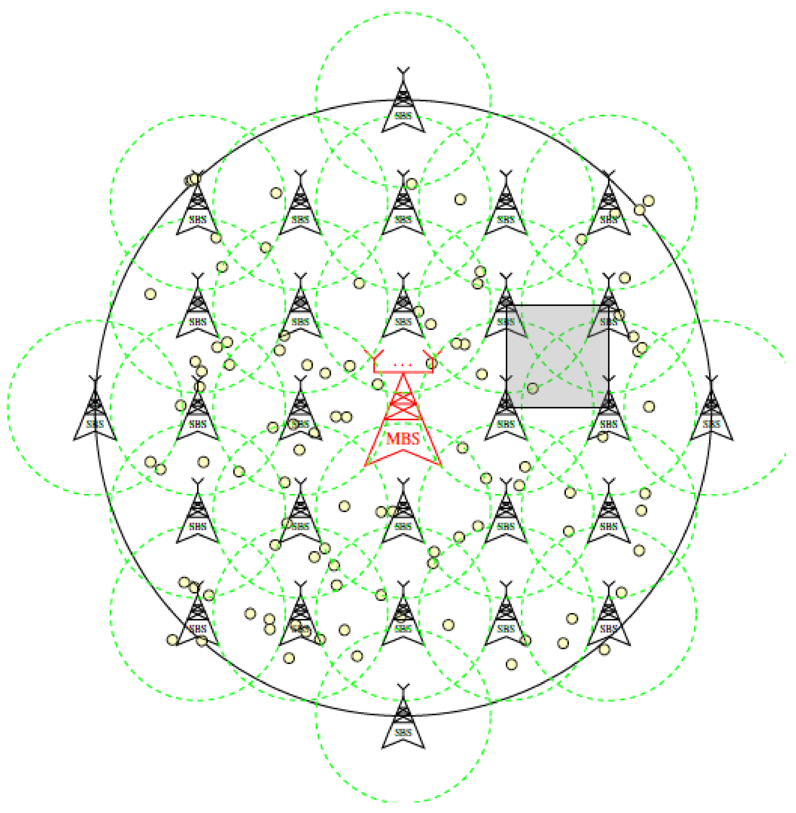
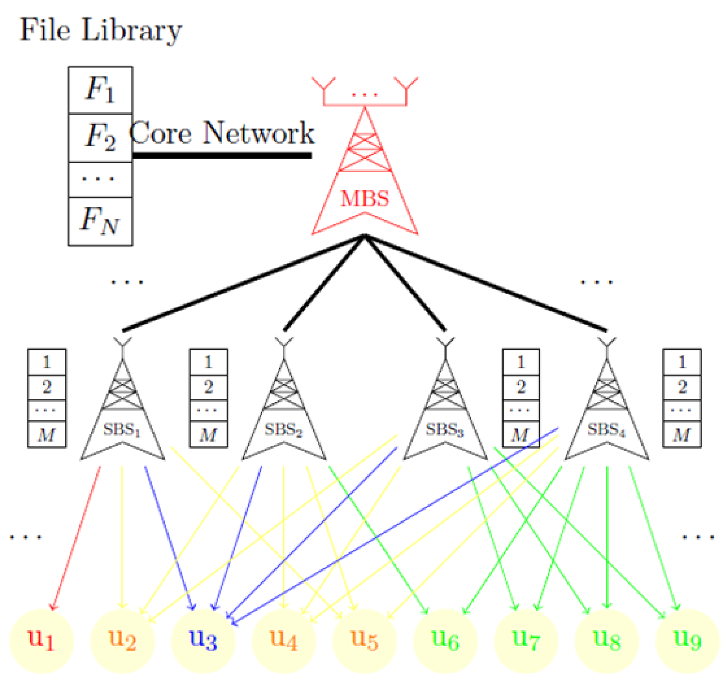
Research Problem

We investigate the problem of optimal MDS-encoded cache placement at the wireless edge to minimize the backhaul rate in heterogeneous networks.



- U wireless users send requests to download files from a macro-cell base station.
- The macro-cell base station (MBS) has access to a library of N files $\mathcal{F} = \{F_1, \dots, F_N\}$, each of size B bits. File F_j is requested with probability p_j with $\sum_{j=1}^{N} p_j = 1$.
- N_{SBS} small-cell base stations with cache of size $M \cdot B$ bits are deployed to serve user requests within overage range r.
- Each user requesting for files in $\mathcal{F}$ is initially served by d_u SBSs depending on its location in the area, in particular, we call γ_i the probability for a user to be served by $d_u = i$ SBSs.
- If the requested file is not completely present in the caches, the MBS has to send the missing data to the user, using the backhaul connection.



MDS Coded Caching Scheme: Definition

1) Placement phase: The MBS creates $E_j = n + (N_{SBS} - 1)m_j$ encoded packets using a MDS(E_j, n) code. The MBS keeps $n - m_j$ encoded packets, and sends the other ones so that each SBS stores m_j unique packets.

2) Delivery phase: A user requesting file F_j contacts $d_u \geq 1$ SBSs, receiving $m_j d_u$ different encoded packets. If $m_j d_u \geq n$, the user can recover the file. Otherwise the MBS sends the remaining $n - m_j d_u \leq n - m_j$ encoded packets. Since the MBS kept $n - m_j$ encoded packets, the user does not receive replicated packets, and can decode the file.

Performance Analysis- Main Results

- MDS coded caching

Proposition The average backhaul rate for an encoded caching placement scheme $\mathcal{C}_{\mathbf{m}}^{\text{MDS}}$ defined by the placement $\mathbf{m} = [m_1 \cdots m_N]$ can be calculated as

$$R_{(\mathcal{C}_{\mathbf{m}}^{\text{MDS}})} = \sum_{d=1}^{S} \sum_{j=1}^{N} \gamma_i p_j \left(1 - \min\left(1, \frac{d m_j}{n}\right)\right), \qquad (1)$$

where $S \leq N_{\text{SBS}}$ is the maximum number of SBSs serving a user.

Idea of Proof Let u be a user served by d_u SBSs. Each SBS stores m_j packets of the file F_j, hence the user can collect $d_u m_j$ different encoded packets. If $d_u m_j \geq n$ the MBS does not send any packet, otherwise $n - d_u m_j$ packets are sent through the backhaul. The rate for user u requesting for file F_j can consequently be calculated as

$$R(F_j) = 1 - \min\left(1, \frac{d_u m_j}{n}\right). \qquad (2)$$

A user has the probability γ_d to be served by d SBSs and each file has a different probability to be requested by a user. Averaging over the request probability distribution $p = [p_1 \cdots p_N]$:

$$R_{(\mathcal{C}_{\mathbf{m}}^{\text{MDS}})} = \sum_{d=1}^{S} \sum_{j=1}^{N} \gamma_d p_j \left(1 - \min\left(1, \frac{d m_j}{n}\right)\right).$$

- Random uncoded caching

To evaluate the gain given by the use of MDS codes, we calculate the backhaul rate of a random caching scheme, where only file fragmentation is exploited.

Proposition The average backhaul rate for a random caching placement scheme $\mathcal{C}_{\mathbf{m}}$ defined by the placement $\mathbf{m} = [m_1 \cdots m_N]$ can be calculated as

$$R_{(\mathcal{C}_{\mathbf{m}})} = \sum_{d=1}^{S} \sum_{j=1}^{N} \gamma_i p_j \left(1 - \frac{m_j}{n}\right)^d. \qquad (3)$$

Given a cache placement $\mathbf{m} = [m_1 \cdots m_N]$, it is preferable in terms of backhaul usage to store MDS coded packets rather than fragments, in particular:

Proposition For any placement $\mathbf{m} = [m_1 \cdots m_N]$, it holds that

$$R_{(\mathcal{C}_{\mathbf{m}}^{\text{MDS}})} \leq R_{(\mathcal{C}_{\mathbf{m}})}.$$

Optimal MDS Coded Caching

Proposition Finding the optimal MDS coded placement scheme $\mathcal{C}_{(\text{opt})}^{\text{MDS}}$ defined by $\mathbf{m}_{(\text{opt})} = [m_1 \cdots m_N]$, which minimizes the average backhaul rate $R_{(\mathcal{C}_{\mathbf{m}}^{\text{MDS}})}$, is a convex optimization problem:

$$\begin{aligned} \min_{q_1,\dots,q_N} & \sum_{d=1}^{S} \sum_{j=1}^{N} \gamma_d p_j (1 - \min(1, d q_j)) \\ \text{s.t. } & \sum_{j=1}^{N} q_j = M \\ & 0 \leq q_j \leq 1 \qquad \forall j \in [1, N] \end{aligned} \qquad (4)$$

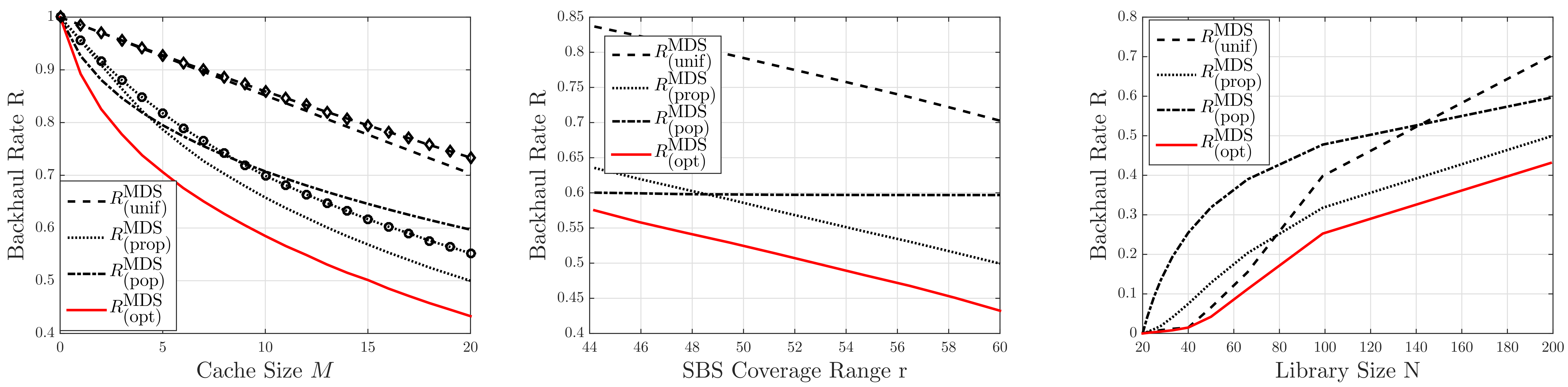
where $q_j = m_j / n$.

Further details

- To be presented at IEEE Globecom 2015, December, San Diego, U.S.A.
- Preprint available at http://arxiv.org/abs/1508.05753

Numerical Illustrations

SBSs are deployed according to a regular grid with a distance $d = 60$ meters between each SBS. The MBS coverage area $\mathcal{R}$ has a radius of $D = 500$ meters. Each SBS has a coverage area of radius r such that $d/\sqrt{2} \leq r \leq d$. We consider the users to be uniformly distributed in $\mathcal{R}$, with density $\rho_d = \rho = 0.05$ users/m^2. These numbers correspond to 316 SBSs being deployed, covering $U = 31,415$ mobile users. The request probability of the files $p = [p_1 \cdots p_N]$ is distributed according to a Zipf law of parameter $\alpha = 0.7$, i.e.,$p_j = \frac{1/j^\alpha}{\sum_j 1/j^\alpha}$.



We compare the optimal achievable backhaul rate $R_{(\text{opt})}^{\text{MDS}}$ to three other practical placement schemes, with and without MDS encoding of the files:

- "most popular" placement $R_{(\text{pop})}$ and $R_{(\text{pop})}^{\text{MDS}}$: each SBS stores the M most popular files;
- uniform placement $R_{(\text{unif})}$ and $R_{(\text{unif})}^{\text{MDS}}$: the file is divided into n fragments, which are placed uniformly at random at each SBS, i.e., each SBS stores M/N fragments of each file;
- proportional placement $R_{(\text{prop})}$ and $R_{(\text{prop})}^{\text{MDS}}$: for each file F_j, n fragments are created. The amount of parts m_j of file F_j stored at the SBSs is proportional to p_j while satisfying the size constraints.